5 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

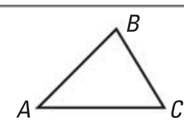
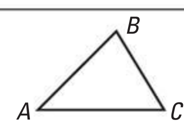
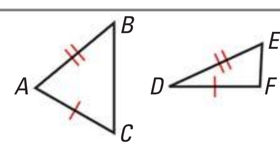
Big Idea 1

Using Properties of Special Segments in Triangles

Special segment	***Properties to remember***
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of △ • center of *circumscribed* circle that passes through 3 vertices of △
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of △ • center of *inscribed* circle that just touches each side of △
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of △
Altitude (perpendicular to side of △ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

Big Idea 2

Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a △ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a △, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a △ are ≅ to two sides of another △, then the △ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.

Big Idea 3

Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

5 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- concurrent, *p. 305*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: A perpendicular bisector is a segment, ray, line, or plane that is perpendicular to a segment at its __?__.

2. WRITING *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

In Exercises 3–5, match the term with the correct definition.

3. Incenter — **A.** The point of concurrency of the medians of a triangle

4. Centroid — **B.** The point of concurrency of the angle bisectors of a triangle

5. Orthocenter — **C.** The point of concurrency of the altitudes of a triangle

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

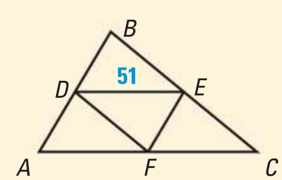
5.1 Midsegment Theorem and Coordinate Proof *pp. 295–301*

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.

By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.



EXERCISES

EXAMPLES 1, 4, and 5 on pp. 295, 297 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

6. If $AB = 72$, find EF.

7. If $DF = 45$, find EC.

8. Graph $\triangle PQR$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.

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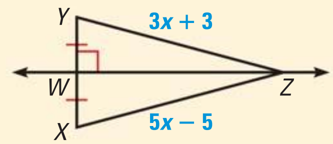
5.2 Use Perpendicular Bisectors *pp. 303–309*

EXAMPLE

Use the diagram at the right to find *XZ*.



$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

$5x - 5 = 3x + 3$ — By the Perpendicular Bisector Theorem, $ZX = ZY$.

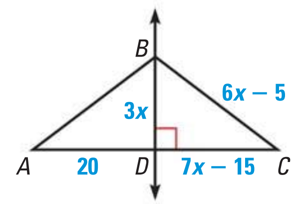
$x = 4$ — Solve for x.

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2 on pp. 303–304 for Exs. 9–11

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.



9. What segment lengths are equal?

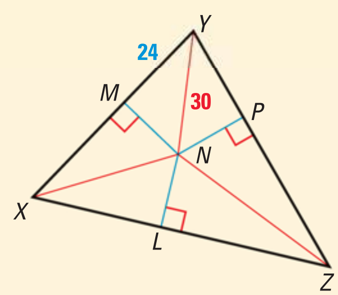
10. What is the value of x?

11. Find AB.

5.3 Use Angle Bisectors of Triangles *pp. 310–316*

EXAMPLE

In the diagram, *N* is the incenter of △*XYZ*. Find *NL*.



Use the Pythagorean Theorem to find *NM* in △*NMY*.

$c^2 = a^2 + b^2$ — Pythagorean Theorem

$30^2 = NM^2 + 24^2$ — Substitute known values.

$900 = NM^2 + 576$ — Multiply.

$324 = NM^2$ — Subtract 576 from each side.

$18 = NM$ — Take positive square root of each side.

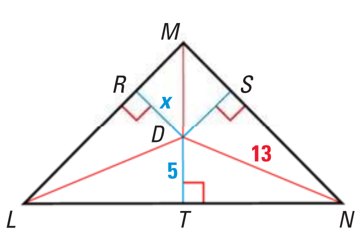
▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter *N* of △*XYZ* is equidistant from all three sides of △*XYZ*. So, because $NM = NL$, $NL = 18$.

EXERCISES

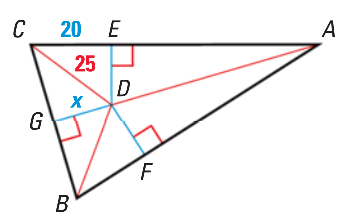
EXAMPLE 4 on p. 312 for Exs. 12–13

Point *D* is the incenter of the triangle. Find the value of *x*.

12.



13.



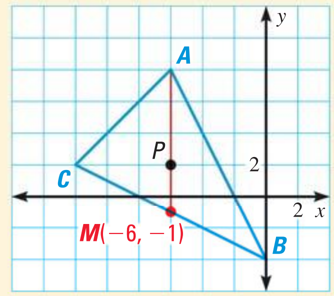
5 CHAPTER REVIEW

5.4 Use Medians and Altitudes

pp. 319–325

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.



Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.

$$M\left(\frac{-12+0}{2}, \frac{2+(-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▶ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 319–321 for Exs. 14–18

Find the coordinates of the centroid D of $\triangle RST$.

14. $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$

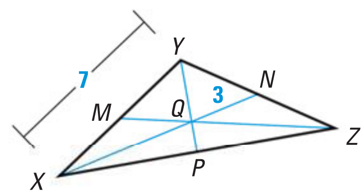
15. $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$

Point Q is the centroid of $\triangle XYZ$.

16. Find XQ.

17. Find XM.

18. Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D.



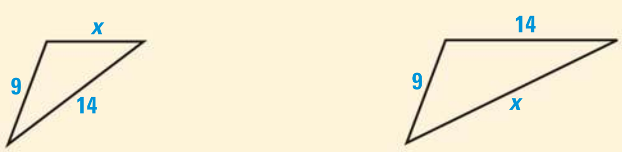
5.5 Use Inequalities in a Triangle

pp. 328–334

EXAMPLE

A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.



$x + 9 > 14$

$x > 5$

$9 + 14 > x$

$23 > x$, or $x < 23$

▶ The length of the third side must be greater than 5 and less than 23.

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 328–330 for Exs. 19–24

Describe **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

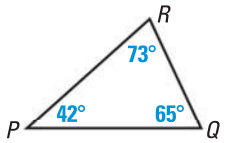
19. 4 inches, 8 inches

20. 6 meters, 9 meters

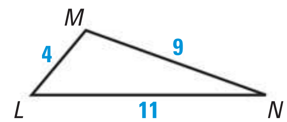
21. 12 feet, 20 feet

List the sides and the angles in order from smallest to largest.

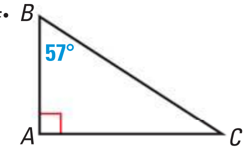
22.



23.



24.

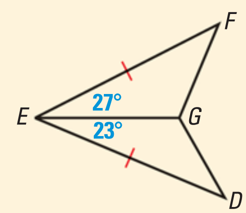


5.6 Inequalities in Two Triangles and Indirect Proof

pp. 335–341

EXAMPLE

How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?



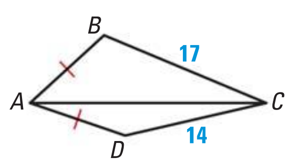
▶ Because 27° > 23°, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle is larger so, by the Hinge Theorem, $FG > DG$.

EXERCISES

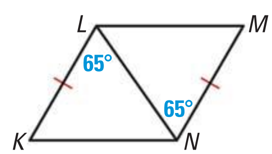
EXAMPLES 1, 3, and 4 on pp. 336–338 for Exs. 25–27

Copy and complete with <, >, or =.

25. $m\angle BAC$ _?_ $m\angle DAC$



26. LM _?_ KN



27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.*

GIVEN ▶ Intersecting lines m and n

PROVE ▶ The intersection of lines m and n is exactly one point.

A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

B. Then there are two lines (m and n) through points P and Q.

C. Assume that there are two points, P and Q, where m and n intersect.

D. It is false that m and n can intersect in two points, so they must intersect in exactly one point.